Dead-Downwind Faster Than The Wind (DDWFTTW) Analysis

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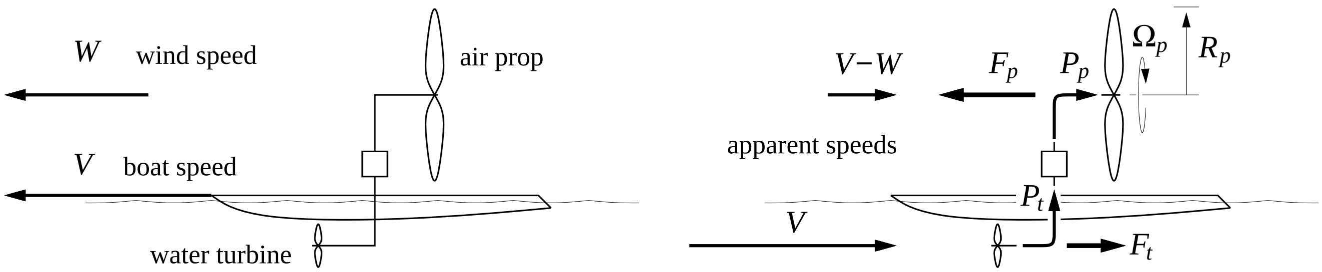
Nomenclature

V	boat speed	ρ	air density
W	wind speed	η_t	water turbine efficiency
F_t	drag force on water turbine	η_g	gearing/transmission efficiency
F_p	thrust force on air prop	η_p	air prop total efficiency
P_t	shaft power of water turbine	η_i	air prop induced efficiency
P_p	shaft power of air prop	η_v	air prop profile efficiency
A_p	air-prop disk area ($= \pi R_p^2$)	$\eta_{\rm swirl}$	air prop swirl efficiency



Velocities

The figure above shows a boat moving with water-speed V, in the same direction as a slower wind speed W. The water turbine therefore sees a water velocity of V, while the air prop sees an air velocity of $V - W$, both opposite the boat motion.

From the definition of turbine and propeller efficiencies:

$$P_t = F_t \, V \, \eta_t \tag{1}$$

$$P_p = F_p \, (V - W)/\eta_p \tag{2}$$

From the definition of the gear/transmission efficiency:

$$P_p = P_t \, \eta_g \tag{3}$$

Substituting for P_p and P_t above gives

$$F_p \, (V - W)/\eta_p = F_t \, V \, \eta_t \, \eta_g \tag{4}$$

$$\text{or} \qquad F_p = F_t \, \frac{V}{V - W} \, \eta_t \, \eta_g \, \eta_p \tag{5}$$

The net thrust available for overcoming the total vehicle water and air drag is therefore

$$F_{\rm net} = F_p - F_t = F_t \left(\frac{V}{V - W} \, \eta_t \, \eta_g \, \eta_p \; - \; 1 \right) \tag{6}$$

which can be positive provided the following holds:

$$\frac{V}{V - W} \, \eta_t \, \eta_g \, \eta_p \; > \; 1 \qquad \text{(requirement for DDWFTTW)} \tag{7}$$

Substituting $F_t = F_p - F_{\mathrm{net}}$ in equation (6) allows solving explicitly for the excess thrust F_{net}.

$$F_{\mathrm{net}} \;=\; F_p \left\{ 1 \;+\; \left[\frac{V}{V\!-\!W} \,\eta_t \,\eta_g \,\eta_p \;-\; 1 \right]^{-1} \right\}^{-1} \tag{8}$$

Air Prop Efficiency Breakdown

The relation (8) above is not useful for initial-design estimation, since $V\!-\!W$ and η_p are both close to zero near the static-thrust condition, and their ratio is crucial. To resolve this problem, the air prop efficiency is broken down into a viscous (profile-drag) efficiency η_v, and an inviscid (or induced) efficiency η_i taken from actuator-disk theory. The latter is modified by including a swirl efficiency η_{swirl} which accounts for non-axial velocities in the slipstream.

$$\eta_p \;=\; \eta_i \,\eta_v \tag{9}$$

$$\eta_i \;=\; \frac{2}{1 \;+\; \left(1 + \dfrac{2F_p}{\rho (V\!-\!W)^2 A_p} \, \dfrac{1}{\eta_{\mathrm{swirl}}}\right)^{1/2}} \tag{10}$$

The F_{net} relation (8) then takes the following equivalent form.

$$F_{\mathrm{net}} \;=\; F_p \left\{ 1 \;+\; \left[\frac{2V \,\eta_t \,\eta_g \,\eta_v}{(V\!-\!W) \;+\; \left((V\!-\!W)^2 + \dfrac{2F_p}{\rho A_p} \, \dfrac{1}{\eta_{\mathrm{swirl}}} \right)^{1/2}} \;-\; 1 \right]^{-1} \right\}^{-1} \tag{11}$$

The very uncertain η_p has now been eliminated, and the four remaining efficiencies can be realistically estimated a priori. Conservative values might be the following.

$$\eta_{\mathrm{swirl}} \;\simeq\; 0.95 \tag{12}$$

$$\eta_v \;\simeq\; 0.90 \tag{13}$$

$$\eta_g \;\simeq\; 0.90 \tag{14}$$

$$\eta_t \;\simeq\; 0.70 \tag{15}$$

The net thrust can now be quickly estimated as a function of the air prop thrust and the few other parameters, which is particularly useful for preliminary design of a DDWFTTW vehicle.

Highly-Loaded Air Prop Limit

When the vehicle is crossing the DDWFTTW threshold, the air prop's relative airspeed $V - W$ is near zero, which means it's operating in "hover" mode. Near this important condition the disk-loading term $2F_p/\rho A_p$ dominates the $V\!-\!W$ terms in relation (11), which can then be simplified as follows.

$$F_{\mathrm{net}} \;=\; F_p \left\{ 1 \;+\; \left[V \, R_p \,\eta_t \,\eta_g \,\eta_v \left(\frac{2\pi\rho}{F_p} \,\eta_{\mathrm{swirl}} \right)^{1/2} \;-\; 1 \right]^{-1} \right\}^{-1} \tag{16}$$

This is simpler and somewhat conservative, so it may be useful for preliminary sizing.

Dimensional Analysis

For optimization purposes, it's useful to introduce the following dimensionless parameters which characterize the operation of any DDW machine.

$$\text{Excess-thrust ratio:} \qquad \mathcal{F} \;=\; \frac{F_{\rm net}}{F_p} \tag{17}$$

$$\text{Apparent velocity ratio:} \qquad Z \;=\; \frac{V-W}{V} \tag{18}$$

$$\text{Modified air prop thrust coefficient:} \qquad C_T' \;=\; \frac{2F_p}{\rho\, V^2\, A_p} \tag{19}$$

The modified thrust coefficient is normalized with the vehicle speed V, rather than the more conventional prop tip speed $\Omega_p R_p$. Ultimately these are related through the turbine's advance ratio and the transmission gearing, although these details can be worked out later and do not need to be considered at this stage. With the definitions above, the net-thrust equation (11) can be put into the following dimensionless form:

$$\mathcal{F} \;=\; \left\{ 1 \;+\; \left[\frac{2\,\eta_t\,\eta_g\,\eta_v}{Z \;+\; \left(Z^2 \;+\; \dfrac{C_T'}{\eta_{\rm swirl}} \right)^{1/2}} \;-\; 1 \right]^{-1} \right\}^{-1} \tag{20}$$

Net Thrust-Drag Balance

The retarding drag force associated with supporting the weight W of the vehicle can be characterized by a resistance coefficient C_r. For a wheeled wehicle this C_r would be the rolling-resistance coefficient, and for a hydrofoil boat this would be the drag/lift ratio. For a buoyancy hull, C_r would be some function of the Froude and Reynolds numbers, whose details are not considered here. Adding on the air resistance, quantified by the air drag area *CDA*, gives the net thrust-drag balance as follows.

$$F_{\rm net} \;=\; W C_r \;+\; \frac{1}{2}\,\rho\,(V-W)^2\,\mathit{CDA} \tag{21}$$

$$\mathcal{F} \;=\; \frac{W}{F_p}\,C_r \;+\; Z^2\,\frac{1}{C_T'}\,\frac{\mathit{CDA}}{A_p} \tag{22}$$

Using this to replace $\mathcal{F}$ in (20) we get an implicit relation for Z.

$$\frac{W}{F_p}\,C_r \;+\; Z^2\,\frac{1}{C_T'}\,\frac{\mathit{CDA}}{A_p} \;=\; \left\{ 1 \;+\; \left[\frac{2\,\eta_t\,\eta_g\,\eta_v}{Z \;+\; \left(Z^2 \;+\; \dfrac{C_T'}{\eta_{\rm swirl}} \right)^{1/2}} \;-\; 1 \right]^{-1} \right\}^{-1} \tag{23}$$

This can be numerically solved for Z if all the other parameters are given. Once Z is known, the corresponding vehicle/wind speed ratio can be determined.

$$\frac{V}{W} \;=\; \frac{1}{1-Z} \tag{24}$$

Numerical Investigation

Equation (23) together with (24) defines the vehicle/wind speed ratio as a function of the following parameters.

$$\frac{V}{W} = f\left(C_T', \eta_{\mathrm{net}}, \eta_{\mathrm{swirl}}, C_r', CDA'\right) \tag{25}$$

where
$$\eta_{\mathrm{net}} = \eta_t\,\eta_g\,\eta_v \tag{26}$$

$$C_r' = \frac{W}{F_p}\,C_r \tag{27}$$

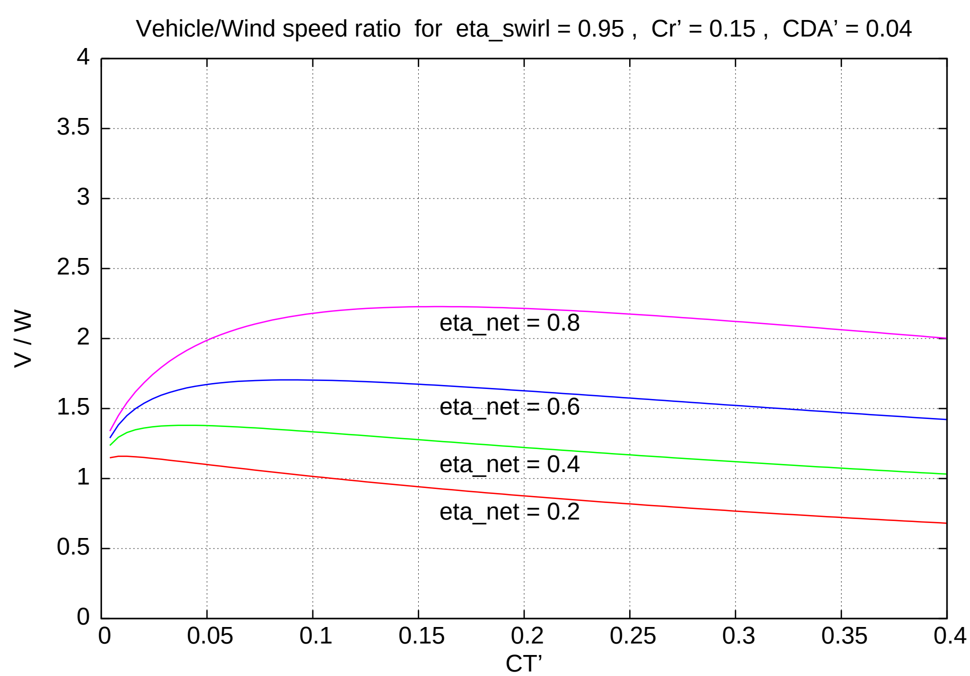
$$CDA' = \frac{CDA}{A_p} \tag{28}$$

With a reasonable constraint on the vehicle length, which is needed to balance the moment of the high thrust line, the ratio W/F_p cannot be made much smaller than 1.0 or so. Hence C_r' is comparable to C_r itself. The drag of the prop tower sets a lower limit on CDA'.

It is useful to now examine the sensitivity of V/W to these parameters. The plot below shows V/W versus C_T', which can be strongly controlled by varying the air-prop diameter, for four values of η_{net}, which depends mainly on the water-turbine and transmission efficiencies. The assumed values for the other parameters might be typical for a well-streamlined water vehicle with a good low-drag hull.

$$\eta_{\mathrm{swirl}} = 0.95 \qquad C_r' = 0.15 \qquad CDA' = 0.04$$

Given that $\eta_t = 0.7$ is not easy for a small vehicle, reaching even $\eta_{\mathrm{net}} = 0.4$ might be challenging. Hence, achieving the DDWFTTW condition $V/W > 1$ would be quite difficult, but possibly doable with careful component design and matching.



Wheeled Vehicles

All the above definitions and equations easily apply to a wheeled ground vehicle, where the water turbine is replaced by the wheels driven by the ground. If the wheel slip is negligible, then we have $\eta_t = 1$. Also, C_r now becomes the conventional rolling-resistance coefficient, which is dramatically smaller than the C_r achievable by a hull. All other quantities should remain roughly the same.

The plot below shows V/W versus C_T' and $\eta_{\rm net}$, with the following assumed remaining parameters.

$$\eta_{\rm swirl} = 0.95 \qquad\qquad C_r' = 0.02 \qquad\qquad CDA' = 0.04$$

Since for this case $\eta_t \simeq 1$, achieving $\eta_{\rm net} = 0.6$ or more is realistic. This confirms that the DDWFTTW condition $V/W > 1$ is achievable with a wheeled vehicle without too much difficulty.