DDWFTTW Power Analysis

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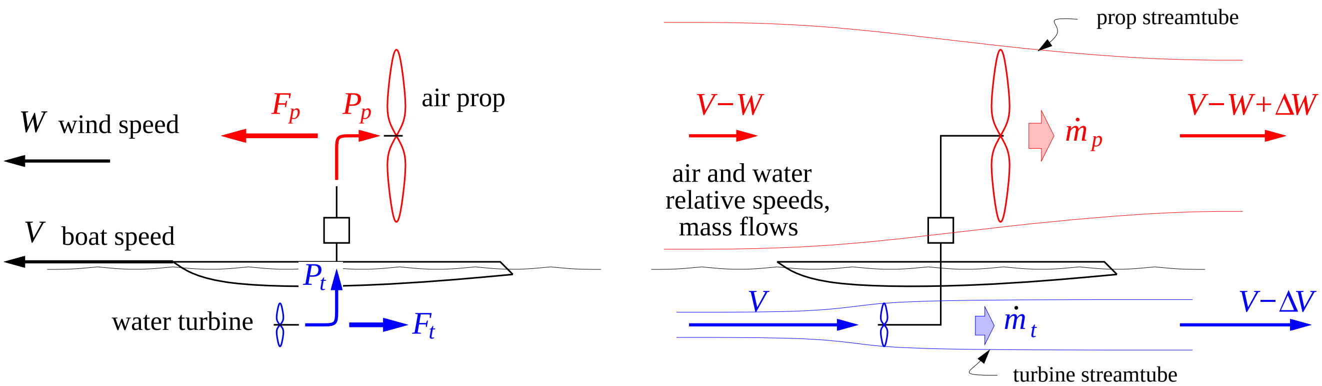
Nomenclature

V	boat speed	W	wind speed
ΔV	water turbine velocity change	ΔW	air prop velocity velocity change
$\dot{m}_t$	water turbine mass flow	$\dot{m}_p$	air prop mass flow
F_t	drag force on water turbine	F_p	thrust force on air prop
P_t	shaft power out of water turbine	P_p	shaft power into air prop



Velocities

The figure above shows a boat moving with water-speed V, in the same direction as a slower wind speed W. The water turbine therefore sees a water velocity of V, while the air prop sees an air velocity of $V-W$, both opposite the boat motion. The downstream velocity changes in the prop and turbine streamtubes are ΔW and ΔV.

Force–Momentum Relations

The force on each rotor is equal to the rate of axial momentum change in each streamtube.

$$F_p = \dot{m}_p \, \Delta[v_p] = \dot{m}_p \, [(V-W+\Delta W) - (V-W)] = \dot{m}_p \, \Delta W \quad (1)$$

$$F_t = -\dot{m}_t \, \Delta[v_t] = -\dot{m}_t \, [(V-\Delta V) - V] = \dot{m}_t \, \Delta V \quad (2)$$

The negative sign for F_t is included because F_t defined positive as shown in the figure, which corresponds to a negative $\Delta[v_{\text{water}}]$.

Power–Kinetic Energy Relations

The shaft power of each rotor is equal to the rate of kinetic energy change in each streamtube.

$$P_p = \tfrac{1}{2}\dot{m}_p \, \Delta\left[v_p^2\right] = \tfrac{1}{2}\dot{m}_p \left[(V-W+\Delta W)^2 - (V-W)^2\right] = \dot{m}_p \left[(V-W)\Delta W + \tfrac{1}{2}\Delta W^2\right] \quad (3)$$

$$P_t = -\tfrac{1}{2}\dot{m}_t \, \Delta\left[v_t^2\right] = -\tfrac{1}{2}\dot{m}_t \left[(V-\Delta V)^2 - V^2\right] = \dot{m}_t \left[V\Delta V - \tfrac{1}{2}\Delta V^2\right] \quad (4)$$

The negative sign for P_t is included for the same reason as for F_t. Using the previous force relations, the power relations can also be given as follows.

$$P_p = F_p \left[(V-W) + \tfrac{1}{2}\Delta W\right] \quad (5)$$

$$P_t = F_t \left[V - \tfrac{1}{2}\Delta V\right] \quad (6)$$

Net Power

The net power available from the prop and turbine combination is

$$P_{\text{net}} \;=\; P_t - P_p \;=\; F_p W \;+\; (F_t - F_p)V \;-\; \tfrac{1}{2}F_t \Delta V \;-\; \tfrac{1}{2}\Delta W \tag{7}$$

If the vehicle is in stready-state operation, the net thrust must be equal to the net drag.

$$F_{\text{net}} \;=\; F_p - F_t \;=\; D \tag{8}$$

The net power then becomes

$$P_{\text{net}} \;=\; P_t - P_p \;=\; F_p W \;-\; DV \;-\; \tfrac{1}{2}F_t \,\Delta V \;-\; \tfrac{1}{2}F_p \,\Delta W \tag{9}$$

The four contributions to the net power (9) can now be readily interpreted:

$F_p W$	power produced by prop thrust moving at wind speed
$-DV$	power lost to drag force moving at boat speed
$-\frac{1}{2}F_t \,\Delta V$	power lost due to kinetic energy deposited in the water
$-\frac{1}{2}F_p \,\Delta W$	power lost due to kinetic energy deposited in the air

In steady-state operation, P_{net} must be sufficiently positive to balance the remaining power losses in the system. The main remaining losses not accounted for are power-transmission losses, profile-drag losses on the prop and turbine blades, and swirl losses in the prop and turbine slipstreams.