# Ecuacion diferencial Circuito RL

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## 1 Circuito RL

Resolver la ecuacion diferencial de un circuito RL con condiciones iniciales $i(t_0) = 0$ y $t_0 = -\frac{\pi}{2w}$ Tenemos: $V cos(wt) = L\frac{di}{dt} + Ri$

$\frac{V}{L} cos(wt) = \frac{d}{dt} i + \frac{R}{L} i$

Factor integracion $= e^{\frac{R}{L}t}$

$e^{\frac{R}{L}t} \frac{d}{dt} i + \frac{R}{L} i e^{\frac{R}{L}t} = \frac{V}{L} cos(wt) e^{\frac{R}{L}t}$

$\frac{d}{dt}[e^{\frac{R}{L}t} i] = \frac{V}{L} cos(wt) e^{\frac{R}{L}t}$

Tenemos que hallar $\frac{V}{L} \int cos(wt) e^{\frac{R}{L}t} dt$ y aplicar las condiciones iniciales. Es decir

$$e^{\frac{R}{L}t} i = \frac{V}{L} \int cos(wt) e^{\frac{R}{L}t} dt \quad (1)$$

Resolver la ecuacion dif $\int cos(wt) e^{\frac{R}{L}t} dt$
Es decir

$$\int cos(wt) e^{\frac{R}{L}t} dt \quad (2)$$

Resolviendo por partes
$u = e^{\frac{R}{L}t}$ y $du = \frac{R}{L} e^{\frac{R}{L}t} dt$ luego
$dv = cos(wt) dt$ y $v = \frac{1}{w} sin(wt)$

$\int cos(wt) e^{\frac{R}{L}t} dt = \frac{1}{w} e^{\frac{R}{L}t} sin(wt) - \frac{R}{Lw} \int sin(wt) e^{\frac{R}{L}t} dt$

y OTRA vez por partes:
$u = e^{\frac{R}{L}t}$ y $du = \frac{R}{L} e^{\frac{R}{L}t} dt$ luego
$dv = sin(wt) dt$ y $v = -\frac{1}{w} cos(wt)$

$\int cos(wt) e^{\frac{R}{L}t} dt = \frac{1}{w} e^{\frac{R}{L}t} sin(wt) - \frac{R}{Lw} [e^{\frac{R}{L}t}(-\frac{1}{w} cos(wt)) + \frac{R}{Lw} \int cos(wt) e^{\frac{R}{L}t} dt]$

$\int cos(wt) e^{\frac{R}{L}t} dt = \frac{1}{w} e^{\frac{R}{L}t} sin(wt) + \frac{R}{Lw^2} e^{\frac{R}{L}t} cos(wt) - \frac{R^2}{L^2 w^2} \int cos(wt) e^{\frac{R}{L}t} dt]$

$[1 + \frac{R^2}{L^2 w^2}] \int cos(wt) e^{\frac{R}{L}t} dt = \frac{1}{w} e^{\frac{R}{L}t} sin(wt) + \frac{R}{Lw^2} e^{\frac{R}{L}t} cos(wt) + C$

$\int cos(wt)e^{\frac{R}{L}t}dt = [\frac{L^2w^2}{L^2w^2+R^2}][\frac{1}{w}e^{\frac{R}{L}t}sin(wt) + \frac{R}{Lw^2}e^{\frac{R}{L}t}cos(wt) + C]$

$\int cos(wt)e^{\frac{R}{L}t}dt = [\frac{L^2w}{L^2w^2+R^2}e^{\frac{R}{L}t}sin(wt) + \frac{LR}{L^2w^2+R^2}e^{\frac{R}{L}t}cos(wt) + C\frac{L^2w^2}{L^2w^2+R^2}]$

$\frac{V}{L}\int cos(wt)e^{\frac{R}{L}t}dt = \frac{V}{L}[\frac{L^2w}{L^2w^2+R^2}e^{\frac{R}{L}t}sin(wt)+\frac{LR}{L^2w^2+R^2}e^{\frac{R}{L}t}cos(wt)+C\frac{L^2w^2}{L^2w^2+R^2}]$

En el paso anterior multiplicamos por $\frac{V}{L}$ para que se parezca a

$$e^{\frac{R}{L}t}i = \frac{V}{L}\int cos(wt)e^{\frac{R}{L}t}dt \qquad (3)$$

y tenemos
$e^{\frac{R}{L}t}i = V[\frac{Lw}{L^2w^2+R^2}e^{\frac{R}{L}t}sin(wt) + \frac{R}{L^2w^2+R^2}e^{\frac{R}{L}t}cos(wt) + C\frac{Lw^2}{L^2w^2+R^2}]$

$i = V[\frac{Lw^2}{L^2w^2+R^2}Ce^{-\frac{R}{L}t} + \frac{Lw}{L^2w^2+R^2}sin(wt) + \frac{R}{L^2w^2+R^2}cos(wt)]$

Aplicando condiciones iniciales tenemos $i(t_0) = 0$ y $t_0 = -\frac{\pi}{2w}$

$i(t_0) = V[\frac{Lw^2}{L^2w^2+R^2}Ce^{-\frac{R}{L}t} + \frac{Lw}{L^2w^2+R^2}sin(w(-\frac{\pi}{2w})) + \frac{R}{L^2w^2+R^2}cos(w(-\frac{\pi}{2w}))]$

$i(t_0) = 0 = V[\frac{Lw^2}{L^2w^2+R^2}Ce^{-\frac{R}{L}t} + \frac{Lw}{L^2w^2+R^2}[-1] + \frac{R}{L^2w^2+R^2}[0]]$

$\frac{Lw^2}{L^2w^2+R^2}Ce^{-\frac{R}{L}t} = \frac{Lw}{L^2w^2+R^2}$

$C = \frac{1}{w}e^{\frac{R}{L}t}$

$i = V[\frac{Lw}{L^2w^2+R^2}e^{\frac{R}{L}t}e^{-\frac{R}{L}t} + \frac{Lw}{L^2w^2+R^2}sin(wt) + \frac{R}{L^2w^2+R^2}cos(wt)]$

$i = V[\frac{Lw}{L^2w^2+R^2}e^{\frac{R}{L}t-\frac{R}{L}t} + \frac{Lw}{L^2w^2+R^2}sin(wt) + \frac{R}{L^2w^2+R^2}cos(wt)]$

$i = V[\frac{Lw}{L^2w^2+R^2}e^0 + \frac{Lw}{L^2w^2+R^2}sin(wt) + \frac{R}{L^2w^2+R^2}cos(wt)]$

$i = V[\frac{Lw}{L^2w^2+R^2} + \frac{Lw}{L^2w^2+R^2}sin(wt) + \frac{R}{L^2w^2+R^2}cos(wt)]$
El resultado del libro es:
$i(t) = V[\frac{wL}{R^2+w^2L^2}e^{(\frac{-t}{\tau}-\frac{\pi}{2wt\tau})} + \frac{R}{R^2+w^2L^2}cos(wt) + \frac{wL}{R^2+w^2L^2}sin(wt)]$

Donde $\tau = L/R$