**3-5.** Differentiating the equation of motion for a simple harmonic oscillator,

$$x = A \sin \omega_0 t \tag{1}$$

we obtain

$$\Delta x = A\omega_0 \cos \omega_0 t \; \Delta t \tag{2}$$

But from (1)

$$\sin \omega_0 t = \frac{x}{A} \tag{3}$$

Therefore,

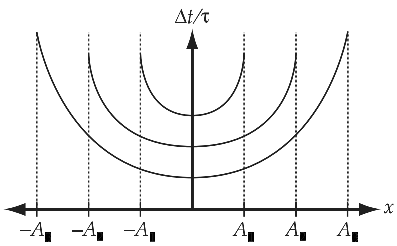
$$\cos \omega_0 t = \sqrt{1-(x/A)^2} \tag{4}$$

and substitution into (2) yields

$$\Delta t = \frac{\Delta x}{\omega_0 \sqrt{A^2 - x^2}} \tag{5}$$

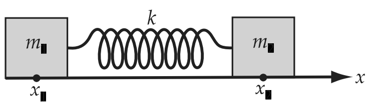
Then, the fraction of a complete period that a simple harmonic oscillator spends within a small interval $\Delta x$ at position $x$ is given by

$$\frac{\Delta t}{\tau} = \frac{\Delta x}{\omega_0 \tau \sqrt{A^2 - x^2}} = \frac{\Delta x}{2\pi \sqrt{A^2 - x^2}} \tag{6}$$




This result implies that the harmonic oscillator spends most of its time near $x = \pm A$, which is obviously true. On the other hand, we obtain a singularity for $\Delta t/\tau$ at $x = \pm A$. This occurs because at these points $\dot{x} = 0$, and (2) is not valid.

**3-6.**




Suppose the coordinates of $m_1$ and $m_2$ are $x_1$ and $x_2$ and the length of the spring at equilibrium is $\ell$. Then the equations of motion for $m_1$ and $m_2$ are

$$m_1 \ddot{x}_1 = -k\left(x_1 - x_2 + \ell\right) \tag{1}$$

$$m_2 \ddot{x}_2 = -k\left(x_2 - x_1 + \ell\right) \tag{2}$$

From (2), we have

$$x_1 = \frac{1}{k}\left(m_2\ddot{x}_2 + kx_2 - k\ell\right) \tag{3}$$

Substituting this expression into (1), we find

$$\frac{d^2}{dt^2}\left[m_1 m_2 \ddot{x}_2 + \left(m_1 + m_2\right)kx_2\right] = 0 \tag{4}$$

from which

$$\ddot{x}_2 = -\frac{m_1 + m_2}{m_1 m_2} kx_2 \tag{5}$$

Therefore, $x_2$ oscillates with the frequency

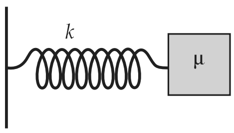
$$\boxed{\omega = \sqrt{\frac{m_1 + m_2}{m_1 m_2} k}} \tag{6}$$

We obtain the same result for $x_1$. If we notice that the reduced mass of the system is defined as

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} \tag{7}$$

we can rewrite (6) as

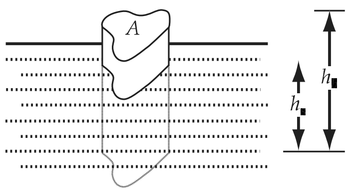
$$\omega = \sqrt{\frac{k}{\mu}} \tag{8}$$




This means the system oscillates in the same way as a system consisting of a single mass $\mu$.

Inserting the given values, we obtain $\mu \simeq 66.7$ g and $\omega \simeq 2.74$ rad $\cdot$ s$^{-1}$.

**3-7.**




Let $A$ be the cross-sectional area of the floating body, $h_b$ its height, $h_s$ the height of its submerged part; and let $\rho$ and $\rho_0$ denote the mass densities of the body and the fluid, respectively.

The volume of displaced fluid is therefore $V = Ah_s$. The mass of the body is $M = \rho A h_b$.